empirical 'extrapolation' formula, which will have the effect of magnifying the accidental errors of observation at the extremities of the table.

The results for the observers with the old Transit Circle from observations made partly through screens in 1900 and 1904 are as follows:—

Mag.	Power.	Pead.
	s	s
0	+0·008	+0·037
1	7	28
2	5	18
3	+0·003	+ ·009
4	...	...
5	− ·004	− ·009
6	− 9	− ·018
7	− 15	− ·026
8	− 21	− ·034
9	− 28	− ·042

The agreement with the present results is as close as could be expected, except in the case of the brighter stars for Power and the fainter stars for Pead. The discordances are doubtless due to the inadequacy of the material employed to correctly determine the magnitude personality at these extreme limits of magnitude, rather than to any real change in the observer's habits during the interval between the two sets of observations.

The plan of these observations was prepared by Sir David Gill, and the observations were well advanced before his departure from the Cape. The observations were completed and the computations made and prepared for press in their present form under the direction of Mr S. S. Hough.

On the Value of the Solar Parallax resulting from the Greenwich Photographs of Eros, 1900–1901.

(*Communicated by the Astronomer Royal.*)

The discussion of the photographs of Eros taken at Greenwich during the opposition of 1900–1901 has now been completed, and a value of the solar parallax has been deduced as resulting from the photographs taken at Greenwich. The plan of the Eros Commission contemplated, ultimately, the combined discussion of the photographs taken at all the co-operating observatories, but there is obviously great difficulty in treating such a mass of heterogeneous material, and it seems desirable, in the first instance, to discuss separately the observations of the individual observatories, and deduce a value of the solar parallax in each case.

With this view the results obtained from the Greenwich photographs are here given. There were in all 197 photographs taken with the Astrographic 13-inch refractor and 153 taken with the Thompson 26-inch refractor between 1900 October 1 and 1901 February 25, but for the determination of the solar parallax the discussion has been confined to the period from 1900 October 14 to 1901 January 18 (151 photographs with the Astrographic and 103 with the Thompson), the material before and after these dates not being suitable for determination of the solar parallax, though useful for the position of Eros.

It may be recalled here that, as explained in a paper on the "New Greenwich micrometer for measurement of photographs of Eros" (*Monthly Notices*, vol. lxiv. p. 633), ten to twelve "reference" stars and six "comparison" stars were measured with Eros on each Astrographic photograph, and the results from these two sets of stars to which Eros was referred have been discussed separately. The "reference" stars selected from M. Loewy's list within 55′ of the plate centre are all brighter than ninth magnitude and therefore brighter than Eros. The "comparison" stars, on the other hand, are of approximately the same brightness as the planet and within 25′ of the plate centre, so as to be well within the field of the Thompson plates for which they served as points of reference.

The determination of the movement of Eros in the interval between the groups of photographs compared, which is essential for the deduction of the parallax, was first undertaken. The measured positions of Eros were compared with tabular places from M. Millosevich's ephemerides (Astrographic Conference, Circular No. 9), using the value 8″·800 for the solar parallax, which was a sufficient approximation for this purpose. The change of error of the ephemeris during the interval between the groups of observations to be compared for parallax was generally considerable (amounting at times to ″·10 per day), but it could be determined with great accuracy. The means of groups of measured positions of Eros were taken so as to give the error of the ephemeris at intervals of about five days; from these the error of the ephemeris was represented by a smooth curve, which represented the observations very satisfactorily.

The correction to the adopted parallax was derived by comparing observations at different hour-angles. It was not usually desirable to make comparisons between groups of observations separated by an interval of more than one day, owing to the large accidental errors introduced by erroneous places of the stars to which Eros was referred. The same reference and comparison stars were used throughout each night, so that in deducing the parallax from comparisons between photographs taken on an evening and the following morning errors in the star-places are eliminated; but in all other comparisons some of the stars used are different for the two groups of plates, and the longer the interval the weaker the connection between the star-places.

From a discussion of the *Right Ascensions* the following results have been derived :—

	Solar Parallax.	*Theoretical Weight.*
Astrographic photographs, Reference Stars,	$8''\cdot793 \pm ''\cdot005$	39·7
,, ,, Comparison Stars,	8·809 ± ·0052	39·7
Thompson photographs, Comparison Stars,	8·800 ± ·0063	31·9
Combined discussion of Astrographic and Thompson photographs, Comparison Stars,	**8·800 ± ·0044**	71·1

The unit of weight used here is the weight of a comparison between one morning and one evening photograph under practically the most favourable conditions, viz. when Eros undergoes a parallactic displacement of 25″ between the two plates.

The difference between the results derived from the Reference and Comparison Stars is of a systematic character. It is found that if plates taken at a large hour-angle are compared with those taken near the meridian (on the same night and with the telescope on the same side of the pier), the comparison stars are displaced relatively to the reference stars, always in the same direction and in the mean by about ″·07 in R.A. It is not clear whether this displacement depends on the comparison stars being fainter than the reference stars, or their being nearer to the centre of the plate. In either case we must suppose that Eros behaves like the comparison stars rather than like the reference stars. For that reason only the results from the comparison stars are used in obtaining the final value of the parallax. It should be noticed that no diminution of the accidental error would be obtained by taking the mean of the results of the comparison and reference stars; for the two determinations are not independent as regards accidental error, depending, as they do, on the same measures of the image of Eros.

The close agreement between the results derived from the Astrographic and Thompson photographs (Comparison stars), and the fact that the same star-places were used in the two cases, seemed to justify a combined discussion of their results, regarding the two sets of places as homogeneous. There was a considerable gain in thus treating them together, as the two instruments had supplemented one another considerably. In this discussion, comparisons extending over more than a day were rigorously excluded. The result given above must be regarded as the principal conclusion of the whole investigation. The results of the separate comparisons are given in Table I., at the end of the paper.

Without much loss of material, the discussion could be rearranged, so that comparisons were only made between the evening and following morning observations. A very considerable reduction in the probable error (inferred from the discordances) took place (notwithstanding the loss of material), owing to the elimination of erroneous star-places. A comparison was also made between the morning and following evening observations, the probable error in this case being more than twice as great.

Thompson and Astrographic Combined.		*Theoretical Weight.*
Evening to Morning comparisons only,	8″·807 ± ″·0036	58·5
Morning to Evening comparisons only,	8·801 ± ·0080	35·4

The probable error for unit weight of an evening to morning comparison is ± ″·028, and for a morning to evening comparison is ± ″·048. The greater part of this difference in the respective probable errors must be attributed to the errors of star-places.

In a former paper, "On the errors of a photographed réseau" (*Monthly Notices*, vol. lxvii. p. 175), it has been shown that the division errors of the central lines of the réseau between which Eros falls affect systematically the deduced value of the solar parallax. Details were there given of the determination of these errors. The corrections there deduced (additional to those which had been provisionally employed), amounting in the mean to + ″·065 for Astrographic photographs, were applied to the positions of Eros before deducing the preceding results. This quantity was very closely confirmed by a discussion of the residuals of the comparison stars, which indicated + ″·068. The probable errors of the two results separately are, however, about ± ″·006 and ± ″·008 respectively. In order that the effect of the uncertainty in the determination of division errors may be estimated, the results obtained before these corrections were applied are given below.

Provisional Results uncorrected for Residual Division Error.

Astrographic photographs, Reference Stars, .	8″·758 ± ″·0054
,, ,, Comparison Stars,	8·774 ± ·0059
Thompson	8·802 ± ·0071

It will be seen that the result from the Thompson photographs is practically unaltered by the application of the new division errors. This is due to the fact that, in the first place, the scale of the Thompson photographs is twice that of the Astrographic, and that the corrections for division error are consequently halved. Further, the accidental variations in the position of Eros relatively to the lines of the réseau were much greater than in the Astrographic photographs, and during part of the opposition the réseau was not reversed when the telescope passed from E to W of the pier. For Astrographic photographs the application of the division correction ″·065 has increased the deduced value of the solar parallax by ″·035.

A discussion was also made using only comparisons between plates taken with the telescope on the same side of the pier, so that the division error of the réseau was eliminated.

Telescope E only.	*Solar Parallax.*	*Theoretical Weight.*
Astrographic reference stars,	8″·748 ± ″·009	12·5
,, comparison ,,	8·783 ± ·009	12·5

The weight of this determination is small, a large part of the material being wasted. It, however, shows very markedly the difference between the reference and comparison stars.

From the *Declinations* of Eros the following values were obtained :—

	Solar Parallax.	*Theoretical Weight.*
Astrographic photographs, reference stars,	$8''\cdot783 \pm ''\cdot018$	7·1
,, ,, comparison ,,	$8''\cdot819 \pm ''\cdot014$	7·1
Thompson photographs ,,	$8''\cdot783 \pm ''\cdot016$	5·5
Mean (comparison stars),	$8''\cdot801 \pm ''\cdot016$	

No special determination of the division errors of the central lines of the réseau was needed for the declinations. The parallax being of the same sign on both sides of the meridian, comparisons were made between plates taken at a large hour-angle E or W and those near the meridian. About the same proportion of low plates were taken with the telescope E and W (réseau direct and reversed) as meridian plates; accordingly the value of the division error of the central lines of the réseau does not affect systematically the deduced parallax. The method employed for deducing the parallax was to divide the observations into groups, each extending over two or three days, and solve each group separately by least squares. The results from the individual groups for Astrographic Comparison Stars are given in Table II.

TABLE I.

Right Ascensions. Astrographic and Thompson combined.

Date.	No. of Plates compared.	Difference of Parallax.	Weight.	Correction to Solar Parallax (adopted value $8''\cdot800$).
1900.		$''$		$''$
Oct. 14–15	4, 1	10	·3	−·045
,, 20	3, 1	13	·4	+·117
, 21	4, 2	23	2·3	+·007
,, 26	6, 4	25	4·8	+·006
,, 27	5, 4	25	4·5	+·038
,, 28–29	2, 5	26	3·1	−·009
Nov. 8–9	2, 2	19	1·2	−·012
,, 9–10	3, 3	27	3·5	+·012
., 10–11	2, 3	27	2·8	+·012
,, 13	5, 5	28	6·3	−·011
,, 14–15	3, 5	24	3·5	−·067
,, 22–23	4, 5	28	5·6	+·009
,, 27	5, 4	15	1·6	−·005
Dec. 6–7	9, 3	16	1·9	+·054
,, 9–10	3, 1	25	1·5	+·011

TABLE I.—*continued.*

Date.	No. of Plates compared.	Difference of Parallax.	Weight.	Correction to Solar Parallax (adopted value 8″·800).
1900.		″		″
Dec. 13	6, 3	21	2·8	− ·044
,, 15–16	6, 6	26	6·5	− ·018
,, 19	5, 5	25	5·0	+ ·003
,, 21	2, 6	27	3·5	+ ·033
,, 28–29	3, 3	12	·7	+ ·033
1901.				
Jan. 5	4, 6	12	1·1	·000
,, 8–9	7, 4	18	2·6	− ·082
,, 13–14	6, 2	22	2·4	+ ·030
,, 15	5, 5	20	3·2	+ ·034
,, 17–18	2, 3	16	1·0	(+ ·277)

Mean Correction + ″·000 ± ″·0044.

The weight is given by the formula $\frac{2mn}{m+n}\left(\frac{\delta\pi}{25''}\right)^2$, where m and n are the numbers of plates in the two groups compared, and $\delta\pi$ is the difference of parallax of Eros between the two groups. No plate is used twice over; *i.e.* if an evening plate is compared with plates taken on the preceding morning, it is not also compared with those taken on the succeeding morning.

The discordant result for Jan. 17–18 has been rejected, as there are several unsatisfactory circumstances in the Thompson photographs on which it depends. On four of the five photographs the images of Eros were noted as difficult to measure, and further, different reference stars had to be used for the two days compared, thus introducing the errors of the star-places which depend on visual observations alone.

TABLE II.

Declinations. Astrographic from Comparison Stars.

Date.	Weight.	Correction to Parallax.
1900.		″
Oct. 14–15	·2	+ ·062
,, 20–21	·4	− ·070
,, 26–27	·6	− ·023
Nov. 9–11	·9	+ ·018
,, 13–15	1·0	+ ·049
,, 22–23	·7	− ·082
Dec. 6–7	·6	+ ·047
,, 13	·5	+ ·040
,, 15–17	·5	+ ·128
,, 19–21	·7	+ ·179
,, 26–29	·3	− ·058

TABLE II.—*continued.*

Date.	Weight.	Correction to Parallax.
1901.		″
Jan. 4–5	·4	+ ·060
,, 8–9	·4	− ·040
,, 14–15	·2	− ·100

Mean Correction + ″·019 ± ″·014.

Royal Observatory, Greenwich:
1907 *April* 9.

The Perturbations of Halley's Comet. By P. H. Cowell and A. C. D. Crommelin.

The differential equations of variation of the four elements, n, e, ϖ, ϵ, the mean motion, eccentricity, longitude of perihelion and epoch are—

$$\frac{1}{m'}\frac{dn}{n} = n_x(a^2X\,du) + n_y(a^2Y\,du)$$

$$\frac{1}{m'}\,de = e_x(a^2X\,du) + e_y(a^2Y\,du)$$

$$\frac{1}{m'}\,ed\varpi = \varpi_x(a^2X\,du) + \varpi_y(a^2Y\,du)$$

$$\frac{1}{m'}\left[d\epsilon - d\varpi\{1 - \sqrt{(1-e^2)}\}\right] = \epsilon_x(a^2X\,du) + \epsilon_y(a^2Y\,du)$$

where

$$n_x = 3\sin u \qquad n_y = -3\sqrt{(1-e^2)}\cos u$$

$$e_x = -\sqrt{(1-e^2)}\sin u\cos u \qquad e_y = \sqrt{(1-e^2)}\{1 - 2e\cos u + \cos^2 u\}$$

$$\varpi_x = -\sqrt{(1-e^2)}\{1 - e\cos u + \sin^2 u\} \qquad \varpi_y = \sin u\cos u - e\sin u$$

$$\epsilon_x = 2\{-\cos u + e(1+\cos^2 u) - e^2\cos u\} \qquad \epsilon_y = 2\sqrt{(1-e^2)}\{-\sin u + e\sin u\cos u\}$$

where $m'X$, $m'Y$ are the components of the disturbing force, the axes of x, y being drawn in the plane of the orbit towards perihelion and parallel to the minor axis respectively.

The quantities n_x, etc., are functions of the eccentricity e, and the eccentric anomaly u only. They are tabulated in Table I. for every odd degree of u, and for the value $\sqrt{(1-e^2)} = ·254000$ or $e = ·967204$. It is clearly unnecessary to tabulate beyond $u = 180°$.

All quantities in the above equations $\frac{dn}{n}$, etc., n_x, etc., $a^2X\,du$, etc., are pure numbers of zero dimensions in space and time. All reference to such arbitrary units as the Earth's mean distance and the mean solar day is thereby avoided.

The first part of Table I. gives the comet's x and y co-ordinates divided by the semi-major axis, and also nt, the comet's mean anomaly in circular measure.